

Math 363/663 Homework 8

due on Sunday 4/19/26

Problem 1. Solve the Laplace equation

$$\begin{cases} \Delta u = 0 & 0 < x < \pi, 0 < y < \pi \\ u(x, 0) = u(x, \pi) = \sin(x) & 0 < x < \pi \\ u(0, y) = u(\pi, y) = \sin(y) & 0 < y < \pi \end{cases}$$

Problem 2. Solve the Laplace equation

$$\begin{cases} \Delta u = 0 & 0 < x < 2, 0 < y < 1 \\ u(x, 0) = 3x, u(x, 1) = \sin(\pi x) + 4x & 0 < x < 2 \\ u(0, y) = 0, u(2, y) = \sin(\pi y) + 2y + 6 & 0 < y < 1 \end{cases}$$

(Hint: Split the boundary conditions appropriately.)

Problem 3. Consider the Neumann problem

$$\begin{cases} \Delta u = 0 & 0 < x < a, 0 < y < b \\ u_y(x, 0) = S(x), u_y(x, b) = N(x) & 0 < x < a \\ u_x(0, y) = W(y), u_x(a, y) = E(y) & 0 < y < b \end{cases}$$

where the normal derivative of u on the boundary of the rectangle is prescribed by the given functions S, N, W, E .

(i) Show that in order for this problem to have a solution, it is necessary to have

$$\int_0^a N(x) dx - \int_0^a S(x) dx + \int_0^b E(y) dy - \int_0^b W(y) dy = 0.$$

(Hint: Write $\int_0^b \int_0^a (u_{xx} + u_{yy}) dx dy = 0$ and apply the fundamental theorem of calculus (or Green's theorem) to the left side.)

(ii) Suppose the functions S, E, W are identically zero. Use separation of variables to find a formal solution of the above problem.