Math 363/663 Homework 7

due on Sunday 3/22/26

Problem 1. Consider the wave equation

$$\begin{cases} u_{tt} = u_{xx} & 0 \le x \le 1,\ -\infty < t < +\infty \\ u(0,t) = u(1,t) = 0 & -\infty < t < +\infty \\ u(x,0) = \sin(3\pi x),\ u_t(x,0) = \sin(2\pi x) & 0 \le x \le 1. \end{cases}$$

Verify directly that the solution $u(x,t)$ given by separation of variables is the same as the one given by d'Alembert's formula.

Problem 2. Find the solution of the wave equation

$$\begin{cases} u_{tt} = u_{xx} & -\infty < x < \infty,\ -\infty < t < +\infty \\ u(x,0) = e^{-x^2},\ u_t(x,0) = \dfrac{2}{x^2+1} & -\infty < x < \infty. \end{cases}$$

What is $\lim_{t\to\infty} u(x,t)$?

Problem 3. Consider the wave equation

$$\begin{cases} u_{tt} = c^2 u_{xx} & -\infty < x < \infty,\ -\infty < t < +\infty \\ u(x,0) = f(x),\ u_t(x,0) = g(x) & -\infty < x < \infty, \end{cases}$$

where the initial profile f and initial velocity g are given smooth functions. Use d'Alembert's formula to verify the following statements:

(i) If f and g are even functions, so is the solution u:

$$u(-x,t) = u(x,t).$$

Similarly, if f and g are odd functions, so is the solution u:

$$u(-x,t) = -u(x,t).$$

(ii) If f and g are both p-periodic, so is the solution u:

$$u(x+p,t) = u(x,t).$$