Math 363/663 Homework 2

due on Friday 2/13/26

Problem 1.

(i) Find the general solution of the first order linear PDE

$$u_x + u_y + u = e^{x+2y}$$

(ii) Find the solution which satisfies the side condition $u(x,0) = 0$.

(iii) How would the answer change if the side condition were $u(x,x) = 0$? What about $u(x,x) = \frac{1}{4}e^{3x}$?

Problem 2. Solve the equation

$$3u_y + u_{xy} = 0.$$

(Hint: Set $v = u_y$ and solve the first order PDE for v. Then find u.)

Problem 3. Solve the first order linear PDE

$$y\,u_x - 4x\,u_y = 2xy$$

which satisfies the side condition $u(x,0) = x^4$. What do the characteristic curves of this PDE look like? (Hint: The short note on the course webpage deals with the case of variable coefficients; see the example there.)