

## Math 363/663 Homework 11

due on Sunday 5/10/26

**Problem 1.** Let  $f(x)$  have the Fourier transform  $F(\omega)$ . Express the Fourier transforms of the functions  $f(2x - 1)$ ,  $e^{i3x}f(x)$ , and  $f''(5x)$  in terms of  $F(\omega)$ .

**Problem 2.** Consider the function

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0. \end{cases}$$

- (i) Find the Fourier transform of  $f$ .
- (ii) Verify that the convolution  $(f * f)(x) = \int_{-\infty}^{\infty} f(y)f(x - y) dy$  is equal to  $xf(x)$ .
- (iii) Find the Fourier transform of  $f * f$  in two different ways: (a) By using the convolution theorem; (b) by using the fact that  $xf(x)$  has the Fourier transform  $iF'(\omega)$ .

**Problem 3.** Verify that the convolution

$$(f * g)(x) = \int_{-\infty}^{\infty} f(y)g(x - y) dy$$

has the following properties:

- (i)  $f * g = g * f$ .
- (ii)  $(f * g)' = f' * g = f * g'$  (here ' means differentiation).
- (iii)  $(f * g) * h = f * (g * h)$ .