

Math 363/663 Homework 10

due on Sunday 5/3/26

Problem 1. Find the Fourier integrals of the functions

$$f(x) = \begin{cases} -|x| + 1 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 1 & 0 < x < 1 \\ -1 & -1 < x < 0 \\ 0 & |x| > 1 \end{cases}$$

For what values of x do the equalities

$$\text{FI}(f)(x) = f(x) \quad \text{and} \quad \text{FI}(g)(x) = g(x)$$

hold?

Problem 2. Use the result of problem 1 to compute the improper integrals

$$\int_0^{\infty} \frac{1 - \cos \omega}{\omega^2} d\omega \quad \text{and} \quad \int_0^{\infty} \frac{(1 - \cos \omega) \sin \omega}{\omega} d\omega$$

Problem 3. Let $a > 0$ be a constant.

(i) Show that the formulas

$$e^{-ax} = \frac{2}{\pi} \int_0^{\infty} \frac{a}{a^2 + \omega^2} \cos(\omega x) d\omega$$

$$e^{-ax} = \frac{2}{\pi} \int_0^{\infty} \frac{\omega}{a^2 + \omega^2} \sin(\omega x) d\omega$$

hold for all $x > 0$.

(ii) Take d/dx of each side of the first formula, assuming differentiation under the integral sign is legitimate. What do you notice? What happens when you attempt a similar differentiation on the second formula?