Math 310 Problem Set 5

9/25/2025

1. True or false? Give a brief proof or a counterexample.

- If $\lim_{n\to\infty} |s_n| = 4$, then either $\lim_{n\to\infty} s_n = 4$ or $\lim_{n\to\infty} s_n = -4$.
- If $\lim_{n\to\infty} s_n = 1$, then $s_n < 2$ for all sufficiently large n.
- If $0.9999 < s_n < 1.0001$ for all $n \geq 500$, then $\lim_{n\to\infty} s_n = 1$.

2. Guess the following limits and use the definition of limit to prove that your guess is correct.

- $\lim\limits_{n\to\infty} \dfrac{2}{n^3}$
- $\lim\limits_{n\to\infty} \dfrac{4n+1}{5n-1}$

3. Suppose $\{s_n\}$ is a sequence that converges to s. If $\{t_n\}$ is another sequence such that $|s_n - t_n| \leq 1/n$ for all $n \in \mathbb{N}$, show that $\{t_n\}$ too converges to s.

4. Suppose s is an accumulation point of a non-empty set $S \subset \mathbb{R}$. Show that there is a sequence $\{s_n\}$ of points in S such that $\lim_{n\to\infty} s_n = s$.

5. (Squeeze Theorem) Let $\{x_n\}, \{y_n\}, \{z_n\}$ be sequences such that $x_n \leq y_n \leq z_n$ for all $n \in \mathbb{N}$. Suppose we know $\lim_{n\to\infty} x_n = L$ and $\lim_{n\to\infty} z_n = L$ (same limit). Show that $\lim_{n\to\infty} y_n = L$.