Math 310 Problem Set 2

9/4/2025

1. Let A, B, C be sets.

(i) Under what condition does the equality $A \setminus (A \setminus B) = B$ hold? Guess the answer using a diagram and then prove it carefully.

(ii) Show that $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

2. Find the following union and intersection of intervals:

$$\bigcup_{n=1}^{\infty} \left(0, 1 - \frac{1}{n}\right) \qquad \text{and} \qquad \bigcap_{n=1}^{\infty} \left(0, 1 + \frac{1}{n}\right).$$

3. Recall that the Cartesian product $\mathbb{Z} \times \mathbb{Z}$ is the set of all ordered pairs (m, n) where both m and n are integers. Let $f : \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be the function defined by $f(n) = (n^2, n)$. Is f injective (one-to-one)? Is it surjective (onto)?

4. Let S be the set of all polynomials of the form $p(x) = ax^2 + bx + c$ and T be the set of all polynomials of the form $q(x) = rx + s$. Here the coefficients a, b, c, r, s can be any real numbers. Let $D : S \to T$ be the "differentiation" mapping defined by $D(p(x)) = p'(x)$. Is D injective? Is it surjective? What is the preimage $D^{-1}(\{x\})$?

5. Suppose $f : A \to B$ and $g : B \to C$ are functions such that the composition $g \circ f : A \to C$ is injective. Is f necessarily injective? What about g? Justify your answers by a proof or counterexample.