Math 152 Second Midterm Review Sheet

4/11/2025

The second midterm will be on Thursday 4/24 during your usual lecture time. It has four problems (one being a few multiple-choice questions) and is 90 minutes long. You are allowed to use a calculator. Here is a list of the topics that you are advised to study:

- Improper integrals
- Application of integration in finding areas and volumes: Suppose we have two functions f, g such that $0 \leq g(x) \leq f(x)$ for $a \leq x \leq b$. Let R be the region bounded from above by the curve $y = f(x)$, from below by the curve $y = g(x)$, on the left by the line $x = a$ and on the right by the line $x = b$. Then

  $*$ The area of R is

  $$A = \int_a^b (f(x) - g(x))\, dx.$$

  $*$ The solid obtained by revolving R about the x-axis has volume

  $$V = \pi \int_a^b (f(x)^2 - g(x)^2)\, dx. \qquad \text{(washer method)}$$

  In the special case $g(x) = 0$, the region R is bounded from below by the x-axis, and the volume formula simplifies to

  $$V = \pi \int_a^b f(x)^2\, dx. \qquad \text{(disk method)}$$

  $*$ Assuming $0 \leq a < b$, the solid obtained by revolving R about the y-axis has volume

  $$V = 2\pi \int_a^b x\,(f(x) - g(x))\, dx. \qquad \text{(cylindrical shells method)}$$

  Again, if $g(x) = 0$ the formula simplifies to

  $$V = 2\pi \int_a^b x\, f(x)\, dx.$$

- In general, the volume of a solid (not necessarily of revolution type) is the integral of its cross-sectional area function:

  $$V = \int_a^b A(x)\, dx.$$

- The arc-length formula: If f is continuously differentiable, the length of the curve $y = f(x)$ between $x = a$ and $x = b$ is

  $$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx.$$

- Solving first order differential equations by separating variables, Newton's law of cooling
- Sequences and their limits, the Squeeze Theorem, increasing and decreasing sequences, bounded monotonic sequences are convergent.

Practice Problems

1. Let p be a fixed number greater than 1. Verify that the improper integral

$$\int_2^\infty \frac{dx}{x(\ln x)^p}$$

is convergent and find its value.

2. Let R be the region in the plane bounded by the curves $y = e^x$ and $y = e^{-x}$, and the line $x = 3$.

(i) Sketch R and find its area.
(ii) Find the volume of the solid generated by rotating R about the x-axis.
(iii) Find the volume of the solid generated by rotating R about the y-axis.

3. Set up an integral for the length of the curve $y = \sin x$ between the points $(0, 0)$ and $(\pi, 0)$. The use your calculator to estimate that integral.

4. Find the general solution to the differential equation

$$\frac{dy}{dx} = xe^{x-y}.$$

Then find the solution that satisfies the initial condition $y(0) = 1$.

5. In each case, find the limit of the given sequence as $n \to \infty$ or show that it does not exist:

$$1 + 2(-1)^n \qquad \frac{1 + 2(-1)^n}{\sqrt{n}} \qquad \frac{n^2}{(5n-1)(n+3)} \qquad \left(1 + \frac{a}{n}\right)^n$$

6. If $\lim_{n\to\infty} \left(\ln(n^2 + 3n - 1) - \ln(cn^2 + 1)\right) = 4$, find c.