Math 152 First Midterm Review Sheet

3/2/2025

The first midterm will be on Thursday 3/13 during your usual lecture time. It has four problems (one being a few multiple-choice questions) and is 90 minutes long. You are allowed to use a calculator. Here is a list of the topics that you are advised to study:

- Inverse functions, derivative of the inverse function

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} \qquad \text{or} \qquad \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

- The natural logarithm $\ln x$ and its basic properties, the derivative formula

$$(\ln x)' = \frac{1}{x} \qquad \text{and more generally} \qquad (\ln u)' = \frac{u'}{u}$$

- The natural exponential function e^x and its basic properties, the derivative formula

$$(e^x)' = e^x \qquad \text{and more generally} \qquad (e^u)' = e^u \cdot u'$$

- General exponential and logarithmic functions a^x and $\log_a x$ and their basic properties, the derivative formulas

$$(a^x)' = a^x \cdot \ln a \qquad \text{and} \qquad (\log_a x)' = \frac{1}{x \ln a}$$

- Logarithmic differentiation
- Exponential growth and decay, doubling time and half-life
- Inverse trigonometric functions $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ and their basic properties, the derivative formulas

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}, \quad (\cos^{-1} x)' = \frac{-1}{\sqrt{1-x^2}}, \quad (\tan^{-1} x)' = \frac{1}{1+x^2}$$

- L'Hospital's rule, its applications in finding limits of indeterminate forms
- The fundamental theorem of calculus, basic integration formulas (for example, review the formulas 1-8, 10, 12, 14, 16, 17 in the table of integrals on reference page 6 in the back of the book)
- Integration by substitution
- Integration by parts
- Trigonometric integrals and trigonometric substitutions
- Partial fractions and integration of rational functions

Practice Problems

1. Verify that the function $f(x) = x^3 + 2x$ has an inverse f^{-1} by showing that f is strictly increasing and therefore one-to-one. Then find the derivative $(f^{-1})'(-3)$.

2. In each case, find the derivative $y' = dy/dx$:

- $y = \ln(x + \sqrt{x})$
- $y = e^{-x} \tan^{-1}(x^2) + \cos^{-1}(5x)$
- $y = (1 - x)^{(x^2)}$

3. Use L'Hospital's rule to find $\lim_{x \to 0} \dfrac{x - \sin x}{x^3}$.

4. The number of bacteria in a lab sample grows exponentially. If $5,000$ bacteria were initially present and $8,000$ were present 10 hours later, how long will it take for the number of bacteria to double?

5. Integrate:

- $\displaystyle\int_0^{2\pi} x^2 \cos x \, dx$ [Hint: Integration by parts twice]
- $\displaystyle\int \left(\sin^2 x + \frac{1}{\sqrt{3 - x^2}} \right) dx$
- $\displaystyle\int \frac{x^3}{\sqrt{x^2 + 4}} \, dx$ [Hint: Trigonometric substitution]
- $\displaystyle\int \frac{x - 4}{x^2 - 5x + 6} \, dx$ [Hint: Partial fractions]

6. Find the area of the region under the curve $y = \dfrac{1}{x^3 + 2x}$ from $x = 1$ to $x = 2$.